

Work Potential Perspective of Engine Component Performance

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There is considerable interest within the propulsion community in applying the concept of thermodynamic work potential as a universal figure of merit for gauging the performance of prime movers. In particular, exergy, gas specific power (GSP), and thrust work potential have shown considerable promise as work potential figures of merit for propulsion system analysis. However, the relationships between these measures of work potential and the classic measures of component performance (component efficiencies) are not widely known. A series of relationships linking component efficiencies to modern measures of work potential are derived. Derivations for a variety of component efficiencies commonly used in aircraft engine analysis are given in terms of all three mentioned work potential measures. Finally, the efficiency-based models are compared and contrasted with modern work potential methods to highlight relative strengths and weaknesses of each.

Nomenclature

C_{FG}	=	gross thrust coefficient
c_p	=	constant pressure specific heat
ex	=	mass-specific exergy
F_G	=	gross thrust
g	=	gravitational acceleration
g_{sp}	=	mass-specific gas power
h	=	mass-specific enthalpy
J	=	work equivalent of heat
LHV	=	lower heating value of fuel
P	=	pressure
PR	=	pressure ratio
q	=	mass-specific heat input
R	=	mass-specific gas constant
s	=	mass-specific entropy
T	=	temperature
V	=	propulsive stream velocity
w	=	mass-specific shaft work
wp	=	mass-specific thrust work potential
γ	=	ratio of specific heats
ε_T	=	turbine effectiveness
η_c	=	compressor adiabatic efficiency
η_{comb}	=	combustion efficiency
η_R	=	inlet pressure recovery
η_T	=	turbine adiabatic efficiency

Subscripts

actual	=	conditions corresponding to real flow process
amb	=	ambient (assumed to be the reference condition)
ideal	=	conditions corresponding to ideal flow process
in	=	mass-specific property flowing into component
loss	=	work potential destroyed (departure from ideal)
out	=	mass-specific property flowing out of component
reject	=	heat rejected due to shaft parasitic losses
0	=	stagnation conditions

Introduction

THERE is a paradigm shift of sorts taking place today in the field of thermodynamics, particularly in thermal systems anal-

ysis. The driving concept behind this change is the idea that every substance has a real and calculable potential to do work and that this work potential is a very powerful tool in understanding the fundamental nature of thermal systems. The concept of work potential is philosophically different and distinct from the classical thermal sciences concept of efficiency in that it gives a holistic view wherein all thermodynamic processes are gauged relative to a single, general figure of merit.

It follows that the concept of thermodynamic work potential holds promise as a universal figure of merit to gauge the performance of propulsion systems. Specifically, the application of these ideas to propulsion system performance analysis leads readily to generalized representations of engine component performance that are directly comparable to one another (unlike component efficiencies). These general representations are a direct measure of the fundamental quantity of interest to propulsion system designers: transfer of work potential.

Presently, expressions relating work potential to component performance are little known, with most of the work in this area focusing on developments of definitions for component effectiveness.¹ The objective of this paper is to establish the links between work potential and the classic efficiency-based figures of merit (FOM) for a variety of standard propulsion system components. Detailed derivations are presented and compared on a component-by-component basis, with emphasis on deriving expressions for transfer of exergy, gas specific power (or gas horsepower), and thrust work potential.

Background

The fundamental principles underlying the concept of thermodynamic work potential have been under development for many decades, starting primarily with the work of Gibbs. These principles have been continuously updated and refined over the years and are today a well-developed field of thermodynamics. The central concept of work potential methods is that every substance has a well-defined work potential stored in it. For instance, a rock at the top of a hill has potential to do work in being moved to the bottom of the hill. Likewise, fuel has a real and measurable quantity of work potential stored in the form of chemical energy of molecular bonds. This quantity is known as exergy.

Exergy is a thermodynamic property describing the maximum theoretical work that can be obtained from a substance in taking it from a given chemical composition, temperature, and pressure to a state of chemical, thermal, and mechanical equilibrium with the environment.² The general definition of exergy is given by

$$ex \equiv h - h_{amb} - T_{amb}(s - s_{amb}) \quad (1)$$

Note that, whereas energy is a conserved quantity, exergy is not and is always destroyed when entropy is produced. Exergy is the

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most comprehensive loss FOM of the three that are examined herein in that it captures the effect of all losses relevant to contemporary propulsive cycles, including nonequilibrium combustion, exhaust heat, and exhaust residual kinetic energy.

Gas specific power (GSP) is defined as the ideal flow-specific shaft power that would be obtained by isentropic expansion of a gas from a specified temperature and pressure through a reversible turbine to a prescribed reference pressure (usually taken to be local atmospheric). The GSP of an ideal, calorically perfect gas at station i is given by³

$$g_{sp,i} = c_p T_i \left[1 - (P_{amb}/P_i)^{(\gamma-1)/\gamma} \right] \quad (2)$$

The temperature at the imaginary expanded condition is a fallout of the isentropic expansion process. Therefore, the definition of GSP is independent of ambient temperature, unlike exergy. GSP is a special case of exergy wherein only mechanical equilibrium with the environment is enforced. It can be thought of as a Brayton FOM because a perfect Brayton cycle will have no loss of GSP, whereas any departure from the ideal Brayton cycle will appear as a loss.

Thrust work potential is equal to ideal thrust multiplied by flight velocity.⁴ Thrust work potential is defined as ideal thrust work that would be obtained in expanding a flow at a given temperature, pressure, and flight velocity to ambient pressure using a thrust nozzle instead of an imaginary turbine. Thrust work potential is a pure jet propulsion FOM because it is a direct index on the ability to produce thrust work directly. In effect, thrust work potential is a measure of ability to project thrust work into the Earth-fixed reference frame and is related to GSP through propulsive efficiency. Thus, thrust work potential is a special case of GSP, and by induction, a special case of exergy.

The preceding three measures of work potential are the basis for the following derivations for component loss as a function of classic component efficiencies. Past derivation and discussion of work potential FOM presented by Ackeret,⁵ Clarke and Horlock,⁶ Curran and Craig,⁷ Riggins,⁸ Roth,⁹ and Roth and Mavris¹⁰ have touched on the idea that component inefficiencies reduce the work potential available in a given cycle from the maximum theoretical to some lesser value. This effect can be thought of as a transfer function that takes maximum theoretical work provided by the cycle into the actual work output provided by the real machine. The difference between the work input and work output is loss and is typically quantified in terms of component efficiencies. These efficiencies are usually defined as a ratio of some actual to ideal quantity, with the exact definition of efficiency varying from component to component. Consequently, one component efficiency is not directly comparable to another.

Work potential, on the other hand, can be used as a universal FOM for thermodynamic performance. Loss of work potential is a component FOM that is directly comparable between components. An added benefit is that loss in work potential provides an absolute measure of thermodynamic cost, which is something that efficiencies do not provide. This paper will explore loss representations of component performance and, where appropriate, discuss some of the prior work that has been done on this topic.

Derivations, Various and Sundry

A useful concept in studying work potential and losses thereof is the work transfer function, defined as the ratio of work potential output to work potential input. This function is bounded by 0 and 1 due to the laws of thermodynamics and can generally be expressed in terms of a few nondimensional parameters. The nondimensional presentation has the benefit that it lays bare the fundamental parameters impacting transfer of work potential. The following sections will present derivations of various work transfer functions for a few of the most prominent component efficiency FOM used in the propulsion industry today.

Loss Due to Nozzle Internal Aerodynamics

Losses in nozzle internal flows are typically quantified in terms of a gross thrust coefficient C_{FG} , defined as the ratio of actual thrust

produced by the nozzle to ideal thrust produced by perfect expansion to ambient pressure. For airbreathing propulsive applications, the prescribed boundary conditions are usually assumed to be known nozzle entrance conditions and exit pressure. Because thrust coefficient is defined as a ratio of actual to ideal thrust, it is directly a thrust work potential FOM. In other words, the thrust work potential transfer function is given by C_{FG} itself,

$$w_{pout}/w_{pin} = C_{FG} \quad (3)$$

An expression for GSP transfer as a function of nozzle thrust coefficient is obtained by noting that ratio of actual to ideal thrust is directly proportional to exit velocity and, therefore, proportional to the square root of actual GSP output to ideal output:

$$C_{FG} \equiv \frac{F_{G,actual}}{F_{G,ideal}} = \frac{\Delta V_{actual}}{\Delta V_{ideal}} = \frac{\sqrt{\Delta g_{spout,actual}}}{\sqrt{\Delta g_{spout,ideal}}} \quad (4)$$

Further, the ideal GSP output is the same as the actual GSP input, so the GSP transfer function in terms of thrust coefficient is easily obtained:

$$g_{spout}/g_{spin} = C_{FG}^2 \quad (5)$$

Loss in GSP due to nozzle thrust coefficient follows from expression (5),

$$g_{sploss} = g_{spin} - g_{spout} = (1 - C_{FG}^2) g_{spin} \quad (6)$$

To derive an expression for exergy transfer as a function of C_{FG} , it is convenient to first note that the transfer function is related to total exergy loss in the nozzle,

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{ex_{loss}}{ex_{in}} = 1 - \frac{ex_{in} - ex_{out}}{ex_{in}} \quad (7)$$

The exergy flowing into and out of the nozzle can be determined using the definition of exergy for a calorically perfect gas given in Eq. (1):

$$ex_{in} = c_p (T_{in} - T_{amb}) - c_p T_{amb} \ln(T_{in}/T_{amb}) + R T_{amb} \ln(P_{in}/P_{amb}) \quad (8)$$

$$ex_{out} = c_p (T_{out} - T_{amb}) - c_p T_{amb} \ln(T_{out}/T_{amb}) + (1/2gJ) V_{out}^2 \quad (9)$$

Note that Eq. (8) presumes that the kinetic energy component of exergy is negligible upstream of the nozzle, whereas Eq. (9) presumes that the pressure component of exergy is zero downstream of the nozzle. If the energy equation is applied to a control volume around the nozzle,

$$c_p T_{out} + (1/2gJ) V_{out}^2 = c_p T_{in}$$

When this equation is substituted into Eq. (9),

$$ex_{out} = c_p (T_{in} - T_{amb}) - c_p T_{amb} \ln(T_{out}/T_{amb}) \quad (10)$$

Substituting Eqs. (8) and (10) into Eq. (7) and noting that

$$c_p = \gamma R/(\gamma - 1), \quad P_{in}/P_{amb} = (T_{in}/T_{amb})^{\gamma/(\gamma-1)} \quad (11)$$

yields a compact expression for exergy transfer in a nozzle:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln(T_{out}/T_{in}) + \ln(T_{in}/T_{amb})}{T_{in}/T_{amb} - 1} \quad (12)$$

T_{out} can be expressed as a function of C_{FG} and T_{in}/T_{amb} by again applying the energy equation and noting that $V_{out} = C_{FG} V_{out,ideal}$:

$$T_{in} = T_{out} + (1/2gJc_p) C_{FG}^2 V_{out,ideal}^2 \quad (13)$$

The second term on the right-hand side is equivalent to the ideal GSP input times the square of the thrust coefficient. When Eq. (3) is substituted into Eq. (13),

$$T_{out} = T_{in} [1 - C_{FG}^2 (1 - T_{amb}/T_{in})] \quad (14)$$

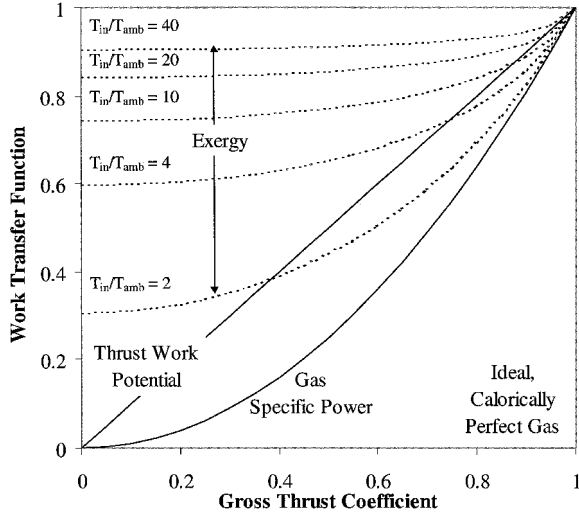


Fig. 1 Work potential transfer as a function of nozzle thrust coefficient.

Finally, substituting Eq. (14) into Eq. (12) yields an expression for exergy transfer as a function of C_{FG} and T_{in}/T_{amb} :

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln[1 - C_{FG}^2(1 - T_{amb}/T_{in})] + \ln(T_{in}/T_{amb})}{T_{in}/T_{amb} - 1} \quad (15)$$

A plot of work transfer function vs nozzle thrust coefficient is given in Fig. 1 for all three work potential FOM. Transfer of thrust work potential has a one-to-one correspondence with thrust coefficient because C_{FG} is a thrust-based FOM. GSP transfer is exceptionally sensitive to nozzle thrust coefficient and is an upper bound on loss in work potential. Exergy is least sensitive to nozzle thrust coefficient because the residual heat in the exhaust efflux increases as C_{FG} decreases, thus providing a recovery effect in which a portion of the lost work potential is reclaimed. (This comes from the fact that exergy presumes that waste heat in the exhaust efflux can be recovered to produce useful work, an unlikely scenario for most gas turbine engines.) However, exergy becomes increasingly sensitive to C_{FG} as T_{in}/T_{amb} gets smaller, eventually approaching the GSP curve as T_{in}/T_{amb} approaches unity.

Loss Due to Pressure Drop

Pressure drops in engine components are usually quantified in terms of percent drop in absolute total pressure relative to the input pressure, $\Delta P/P_{in}$. Exergy loss due to a pressure drop can be calculated by application of the Gouy–Stodola lost work theorem, which states that the loss in exergy work potential is equal to the product of the reference temperature and the entropy generated (see Ref. 2):

$$\left(\frac{\text{lost work}}{\text{unit mass}}\right) = ex_{loss} = T_{amb} \Delta s = T_{amb} R \ln\left(\frac{P_{out}}{P_{in}}\right) \quad (16)$$

Δs is the change in entropy caused by the pressure drop, P_{in} and P_{out} are the pressure before and after the pressure drop, respectively, and R is the mass-specific gas constant. Next, expressing this equation in terms of $\Delta P/P_{in}$ yields

$$ex_{loss} = T_{amb} R \ln(1 - \Delta P/P_{in}) \quad (17)$$

Recall that the expression for the exergy transfer function in terms of exergy loss is

$$ex_{out}/ex_{in} = 1 - ex_{loss}/ex_{in} \quad (7)$$

Thus, combining equations gives

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{RT_{amb} \ln(1 - \Delta P/P_{in})}{ex_{in}} \quad (18)$$

Substituting in Eq. (1) into the denominator and canceling RT_{amb} from the numerator and denominator yields an expression for exergy

transfer as a function of pressure drop and nondimensional inlet pressure and temperature:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln(1 - \Delta P/P_{in})}{[\gamma/(\gamma - 1)][T_{in}/T_{amb} - 1 - \ln(T_{in}/T_{amb})] + \ln(P_{in}/P_{amb})} \quad (19)$$

An analogous expression for GSP transfer across pressure drops can be derived by applying the expression for GSP of an ideal, calorically perfect gas [Eq. (2)] to the inlet and outlet stations to obtain two equations:

$$g_{sp_{in/out}} = c_p T_{in/out} [1 - (P_{in/out}/P_{amb})^{(1-\gamma)/\gamma}] \quad (20)$$

Note that P_{out} can be expressed as

$$P_{out} = P_{in}(1 - \Delta P/P_{in}) \quad (21)$$

Forming the ratio $g_{sp_{in}}/g_{sp_{out}}$ using Eqs. (20) and substituting for P_{out} yields

$$\frac{g_{sp_{out}}}{g_{sp_{in}}} = \frac{1 - [(P_{in}/P_{amb})(1 - \Delta P/P_{in})]^{(1-\gamma)/\gamma}}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}} \quad (22)$$

Thus, GSP transfer is a function of the ratio of specific heats, the nondimensional inlet pressure, and the pressure drop.

Exergy and GSP transfer are plotted as a function of component pressure loss in Fig. 2. The solid lines show GSP transfer over a range of nondimensional inlet pressures. Note that GSP loss is highly sensitive to pressure loss at low inlet pressures, but becomes less sensitive as the inlet pressure increases. The dashed lines show exergy transfer for a range of nondimensional inlet temperatures at a single nondimensional inlet pressure of two. A family of such curves exists for each value of nondimensional inlet temperature. Note that as inlet temperature increases, the exergy transfer function becomes insensitive to pressure drops. Also note that the pressure drop curves for P_{in}/P_{amb} end at 50% because higher pressure drops imply outlet pressures below ambient. Exergy transfer is substantially the same as GSP transfer when the nondimensional inlet temperature is one.

It was implied in Eq. (4) that the thrust work potential transfer function is related to the GSP transfer function via the relation

$$wp_{out}/wp_{in} = \sqrt{g_{sp_{out}}/g_{sp_{in}}} \quad (23)$$

This same relationship will be used in all following derivations as the basis to convert GSP transfer functions into thrust work potential transfer functions.

Contours of thrust work potential transfer and contours of constant GSP transfer are plotted in Fig. 3. Note that thrust work potential is generally less sensitive to pressure losses than is GSP,

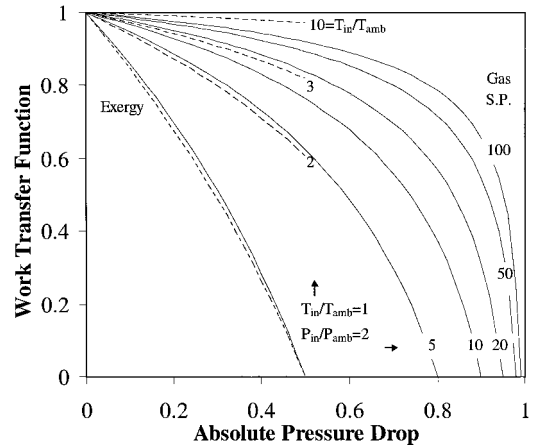


Fig. 2 Exergy and GSP work potential transfer as a function of component pressure drop.

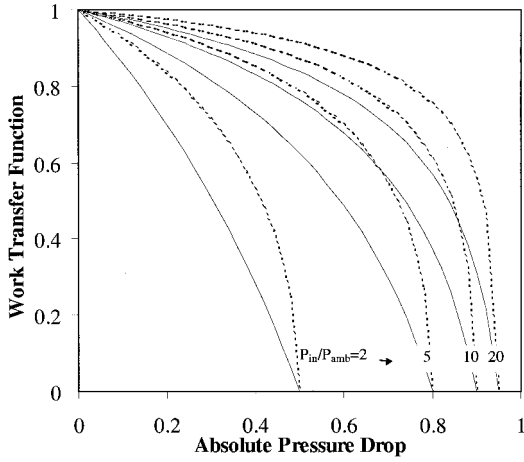


Fig. 3 Comparison of thrust work potential (---) and GSP (—) work transfer as a function of pressure drop.

especially at high inlet pressure. This result may at first seem counterintuitive given that a portion of the GSP must inevitably be lost in the form of exhaust residual kinetic energy (propulsive efficiency effects). The answer lies in the realization that thrust work potential does not bookkeep exhaust residual kinetic energy as useful work potential.³ Therefore, the residual kinetic energy loss is immaterial as far as ratios of losses are concerned. This observation is reflected in Fig. 3.

Loss Due to Inlet Pressure Recovery

Expressions for work potential transfer as a function of inlet pressure recovery can be obtained directly from the equations developed in the preceding section. From the definition of inlet pressure recovery,

$$P_{0,\text{out}} = \eta_R P_{0,\text{in}} \quad (24)$$

Substituting this expression in the earlier developed equations readily yields formulas for work transfer as a function of inlet recovery.

$$\frac{ex_{\text{out}}}{ex_{\text{in}}} = 1 - \frac{\ln(\eta_R)}{[\gamma/(\gamma-1)][T_{\text{in}}/T_{\text{amb}} - 1 - \ln(T_{\text{in}}/T_{\text{amb}})] + \ln(P_{\text{in}}/P_{\text{amb}})} \quad (25)$$

$$\frac{g_{\text{sp,out}}}{g_{\text{sp,in}}} = \frac{1 - [(P_{\text{in}}/P_{\text{amb}})\eta_R]^{(1-\gamma)/\gamma}}{1 - (P_{\text{in}}/P_{\text{amb}})^{(1-\gamma)/\gamma}} \quad (26)$$

Loss Due to Compressor Adiabatic Efficiency

To derive an expression for exergy transfer as a function of compressor adiabatic efficiency η_c , recall that the exergy transfer function is related to total exergy loss via

$$\frac{ex_{\text{out}}}{ex_{\text{in}}} = 1 - \frac{ex_{\text{loss}}}{ex_{\text{in}}} = 1 - \frac{ex_{\text{in}} - ex_{\text{out}}}{ex_{\text{in}}} \quad (7)$$

For a calorically perfect ideal gas, the exergy loss can be determined using the Gouy–Stodola lost work theorem already mentioned,

$$ex_{\text{loss}} = T_{\text{amb}} \Delta s \quad (27)$$

where Δs is the entropy generation across the compressor. Δs can be expressed in terms of temperature and pressure for an ideal, calorically perfect gas using the TdS relations. Furthermore, the definition of compressor adiabatic efficiency is the ratio of ideal work to actual work required to achieve a given pressure ratio:

$$\eta_c = \frac{h_{\text{out,ideal}} - h_{\text{in}}}{h_{\text{out}} - h_{\text{in}}} \approx \frac{T_{\text{out,ideal}} - T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} \quad (28)$$

When these four relations are combined, it can be shown that

$$\frac{ex_{\text{out}}}{ex_{\text{in}}} = 1 - \frac{\eta_c T_{\text{amb}} \ln(T_{\text{out}}/T_{\text{out,ideal}})}{T_{\text{out,ideal}} - T_{\text{in}}} \quad (29)$$

The quantity inside the parentheses can be expressed in terms of compressor efficiency by rearranging the terms in the definition of η_c :

$$T_{\text{out}}/T_{\text{out,ideal}} = 1/\eta_c + (T_{\text{in}}/T_{\text{out,ideal}})(1 - 1/\eta_c) \quad (30)$$

Furthermore, the ratio of inlet to ideal outlet temperature can be expressed in terms of compressor pressure ratio (PR) using isentropic compressible flow relations:

$$T_{\text{out,ideal}}/T_{\text{in}} = \text{PR}^{(\gamma-1)/\gamma} \quad (31)$$

Substituting Eqs. (30) and (31) into (29) and rearranging yields an expression for compressor exergy transfer (also known as compressor effectiveness) as a function of compressor adiabatic efficiency, PR, and nondimensional inlet temperature:

$$\frac{ex_{\text{out}}}{ex_{\text{in}}} = 1 - \frac{\eta_c \ln[\text{PR}^{(1-\gamma)/\gamma} (1 - 1/\eta_c) + 1/\eta_c]}{(T_{\text{in}}/T_{\text{amb}})(\text{PR}^{(\gamma-1)/\gamma} - 1)} \quad (32)$$

This equation can be used to create a plot of exergy transfer as a function of compressor adiabatic efficiency, as shown in Fig. 4. Figure 4 shows two families of curves. The solid lines show exergy transfer for a range of nondimensional inlet temperatures with PR fixed at two. The dashed lines show exergy transfer for a range of PRs with nondimensional inlet temperature fixed at one. Note that the exergy transfer function approaches unity as compressor adiabatic efficiency approaches unity. Furthermore, note that the exergy transfer is never less than the compressor efficiency itself, a well-known result.¹¹ It is clear from Fig. 4 that increasing the nondimensional inlet pressure decreases the relative proportion of exergy loss due to compression, as does increasing the compressor PR. Conversely, it is clear from Fig. 4 that low-pressure-ratio devices (fans and propellers) will be heavily impacted by exergy losses if the compression system does not have a high adiabatic efficiency.

A surprising feature of Fig. 4 is the manner in which exergy transfer approaches unity as compressor efficiency approaches zero. This trend is quite counterintuitive, but can be explained as follows. The definition of compressor adiabatic efficiency is the ratio of ideal to actual work required to achieve a given PR. Therefore, as compressor efficiency approaches zero, the work required to achieve the PR becomes exorbitant, thus implying that compressor discharge temperature is also very high. Therefore, the exergy content of the compressor discharge at very low η_c is primarily made up of heat energy instead of compression work. In effect, at high compressor

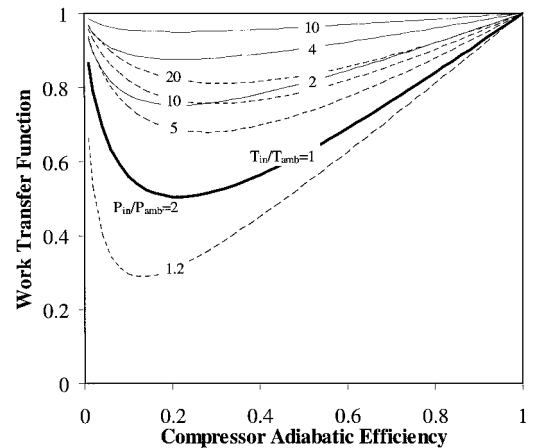


Fig. 4 Exergy transfer as a function of compressor efficiency for a range of pressure ratios (---) and temperature ratios (—).

efficiency, the primary mechanism of exergy transfer is via compression work. At very low efficiency, the compressor is a flow heater, delivering exergy transfer via high discharge temperature.

An expression for GSP transfer as a function of compressor adiabatic efficiency can be derived by substituting the expression for GSP of an ideal gas [Eq. (2)] into the definition of the GSP transfer function.

$$\frac{g_{sp_{out}}}{g_{sp_{in}}} = \frac{c_p T_{out} [1 - (P_{out}/P_{amb})^{(1-\gamma)/\gamma}]}{c_p T_{in} [1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}] + w_{in}} \quad (33)$$

where w_{in} is the net shaft work into the compressor. This shaft work can be related to the ideal (isentropic) work through the definition of compressor efficiency,

$$w_{in} = w_{ideal}/\eta_c = (c_p T_{out,ideal}/\eta_c) [1 - (P_{out}/P_{in})^{(1-\gamma)/\gamma}] \quad (34)$$

When substituted and rearranged

$$\begin{aligned} \frac{g_{sp_{out}}}{g_{sp_{in}}} &= \frac{(T_{out}/T_{in}) [1 - (P_{out}/P_{amb})^{(1-\gamma)/\gamma}]}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma} + (T_{out,ideal}/T_{in}) [1 - (P_{out}/P_{in})^{(1-\gamma)/\gamma}]} \quad (35) \end{aligned}$$

The ratio of T_{out}/T_{in} can be expressed in terms of compressor efficiency and $T_{out,ideal}/T_{in}$ using the definition of compressor efficiency. Furthermore, $T_{out,ideal}/T_{in}$ can be expressed as a function of PR using Eq. (31). Substituting into Eq. (35) yields an expression for GSP transfer as a function of nondimensional inlet pressure, PR, gamma, and compressor efficiency:

$$\begin{aligned} \frac{g_{sp_{out}}}{g_{sp_{in}}} &= \frac{[1 + (1/\eta_c)(PR^{(\gamma-1)/\gamma} - 1)] \{1 - [PR(P_{in}/P_{amb})]^{(1-\gamma)/\gamma}\}}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma} + (1/\eta_c)PR^{(\gamma-1)/\gamma} [1 - PR^{(1-\gamma)/\gamma}]} \quad (36) \end{aligned}$$

If the inlet pressure is assumed to be ambient pressure, this expression reduces to

$$\frac{g_{sp_{out}}}{g_{sp_{in}}} = \frac{\eta_c + PR^{(\gamma-1)/\gamma} - 1}{PR^{(\gamma-1)/\gamma}} \quad (37)$$

Note that this is a linear function of compressor adiabatic efficiency.

Equation (37) can be used to create a plot of GSP transfer as a function of compressor adiabatic efficiency, as shown in Fig. 5. The dashed lines in Fig. 5 are the exergy transfer contours as a function of PR, repeated from Fig. 4. The solid lines are GSP transfer as a function of compressor efficiency for the same range of PR as was

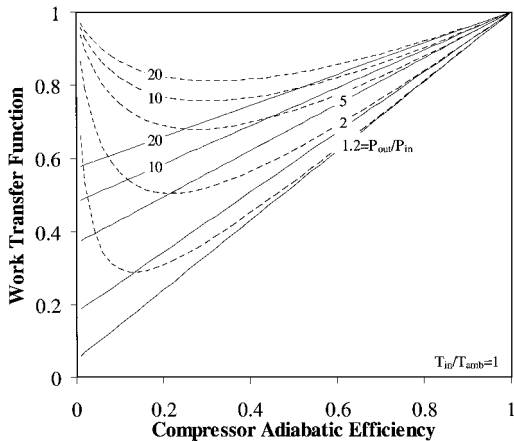


Fig. 5 GSP (—) and exergy transfer (---) as a function of compressor efficiency for a range of PRs.

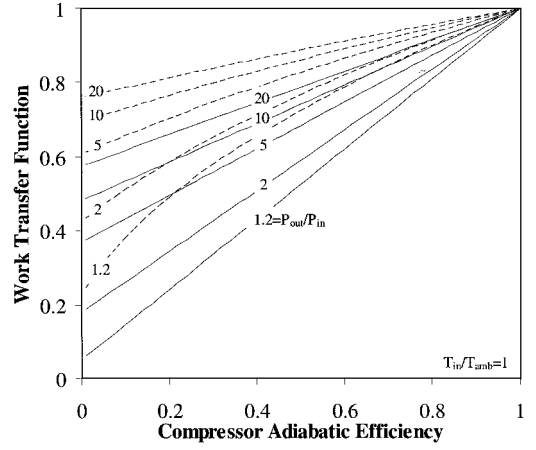


Fig. 6 Comparison of GSP work transfer (—) against thrust work potential transfer (---) for a range of PRs.

plotted for exergy. Note that the GSP and exergy transfer converge at high efficiency, but diverge as efficiency decreases. Whereas exergy transfer ultimately recovers to one as efficiency approaches zero, GSP transfer can only recover a small portion of the heat addition in the form of usable work. Also, note that GSP work transfer becomes increasingly sensitive to compressor efficiency as PR decreases. In the limit as PR approaches unity, the GSP work transfer becomes equal to compressor adiabatic efficiency.

Finally, the impact of compressor efficiency on thrust work potential is easily estimated by taking the square root of the GSP transfer function. When it is presumed that the inlet pressure is once again the same as ambient, the expression for thrust work transfer is

$$\frac{w_{p_{out}}}{w_{p_{in}}} = \sqrt{\frac{\eta_c + PR^{(\gamma-1)/\gamma} - 1}{PR^{(\gamma-1)/\gamma}}} \quad (38)$$

Figure 6 shows a plot comparing GSP contours shown previously against thrust work potential contours (dashed lines). Note that the thrust work transfer function is always higher than the GSP transfer, though one should always bear in mind that total thrust work will always be less than total GSP.

It is clear, based on the equations developed in this subsection, that compressor performance can be represented in terms of work potential and loss thereof. Given this idea, one is naturally led to inquire about the practicality of building a compressor loss map analogous to the standard compressor efficiency maps used today. Paulus et al.¹² have recently published investigations into this subject, though using entropy as an index of compressor loss instead of GSP or exergy. Their results show that such component performance representations are indeed possible and possibly even somewhat more in tune with the fundamental physics driving compressor performance than are existing methods.

Loss Due to Turbine Adiabatic Efficiency

Expressions for work transfer as a function of turbine adiabatic efficiency can be derived in parallel fashion to those for compressor work transfer. When started with the definition of second law effectiveness for a turbine,¹¹

$$\varepsilon_T \equiv \frac{ex_{out}}{ex_{in}} = \frac{w_{out}}{ex_{in} - ex_{out}} = 1 / \left[1 + T_{amb} \frac{\ln(T_{out}/T_{out,ideal})}{\eta_T (T_{in} - T_{out,ideal})} \right] \quad (39)$$

The term inside the natural logarithm can be expressed as a function of turbine PR (defined as P_{in}/P_{out} for a turbine) and turbine efficiency. With some rearrangement and substitution, this readily yields an expression for turbine exergy transfer as a function of turbine adiabatic efficiency, turbine PR, nondimensional inlet temperature, and gamma:

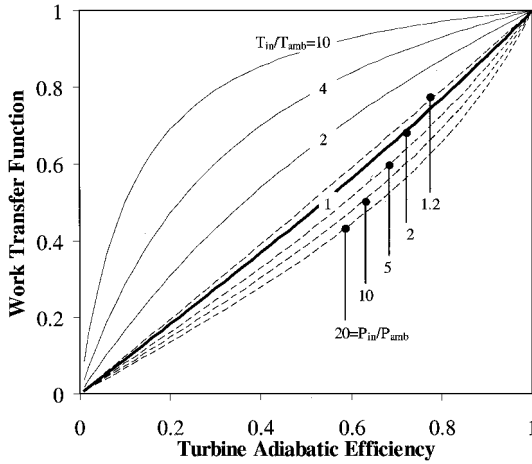


Fig. 7 Exergy transfer as a function of turbine adiabatic efficiency for a range of inlet pressures and temperatures.

$$\frac{ex_{out}}{ex_{in}} = \left\{ 1 + \frac{\ell_n [PR^{(\gamma-1)/\gamma} (1 - \eta_T) + \eta_T]}{(T_{in}/T_{amb}) \eta_T (1 - PR^{(1-\gamma)/\gamma})} \right\}^{-1} \quad (40)$$

The impact of turbine exergy transfer as a function of adiabatic efficiency is shown in Fig. 7 for a range of inlet pressures and temperatures. Note that, as inlet temperature increases exergy transfer becomes less sensitive to turbine efficiency. Conversely, as turbine PR increases, exergy transfer becomes increasingly sensitive to turbine efficiency.

An expression for GSP transfer can be derived using an approach directly paralleling that used earlier for compressor GSP transfer,

$$\begin{aligned} \frac{g_{spout}}{g_{spin}} &= \frac{\{1 - \eta_T [1 - PR^{(1-\gamma)/\gamma}]\} \{1 - [(P_{in}/P_{amb})(1/PR)]^{(1-\gamma)/\gamma}\}}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}} \\ &+ \eta_T PR^{(1-\gamma)/\gamma} \end{aligned} \quad (41)$$

If P_{out} is assumed to be P_{amb} , the preceding expression reduces to

$$g_{spout}/g_{spin} = \eta_T PR^{(1-\gamma)/\gamma} \quad (42)$$

This is again a linear function of turbine adiabatic efficiency as it

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{[(f/a)(LHV)/c_p T_{amb}](1 - \eta_{comb})}{[T_{in}/T_{amb} - 1 - \ell_n(T_{in}/T_{amb})] + \ell_n(P_{in}/P_{amb}) + [(f/a)(LHV)/c_p T_{amb}]} \quad (52)$$

was in the case of compressor GSP transfer. Once again, thrust work potential transfer is the square root of this quantity.

Loss Due to Incomplete Combustion

Loss due to incomplete combustion is typically quantified in terms of combustion efficiency, defined as actual heat released by the combustion process divided by the heat release of the ideal combustion process, given by the lower heating value of the fuel times the fuel/air ratio (q_{ideal}):

$$\eta_{comb} = q_{actual}/q_{ideal} \quad (43)$$

An expression for GSP transfer as a function of combustion efficiency is easily derived by applying Eq. (2) and recognizing that

$$g_{spout}/g_{spin} = 1 - g_{sploss}/g_{spin} \quad (44)$$

If g_{spin} is taken to be the GSP flowing into the combustor plus the ideal GSP content of the fuel injected, expression (44) becomes

$$\frac{g_{spout}}{g_{spin}} = 1 - \frac{c_p (T_{out,ideal} - T_{out}) [1 - (P_{out}/P_{amb})^{(1-\gamma)/\gamma}]}{c_p T_{out,ideal} [1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}]} \quad (45)$$

If the combustion process is assumed to occur at constant pressure, it can be shown that GSP transfer becomes

$$\frac{g_{spout}}{g_{spin}} = 1 - \frac{T_{out,ideal} - T_{out}}{T_{out,ideal}} = \frac{(f/a)(LHV)\eta_{comb} + T_{in}c_p}{(f/a)(LHV) + T_{in}c_p} \quad (46)$$

Therefore, GSP is a linear function of combustion efficiency. Thrust work transfer is the square root of the preceding quantity. Note that this derivation implicitly assumed constant specific heats, a very restrictive assumption for most practical applications.

An expression for loss in GSP can be derived by noting that the rise in GSP across the combustor, assuming no pressure drop, can be approximated as

$$\Delta g_{spideal} = c_p \Delta T_{ideal} [1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}] \quad (47)$$

where ΔT_{ideal} is the ideal combustor temperature rise. Now, with use made of the definition of combustion efficiency once again,

$$\eta_{comb} = Q/Q_{ideal} \approx \Delta T_{actual}/\Delta T_{ideal} \quad (48)$$

and with substitution of this back into the expression for ideal available energy rise,

$$\Delta g_{spideal} = (c_p \Delta T_{actual}/\eta_{comb}) [1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}] \quad (49)$$

Thus, loss in available energy due to incomplete combustion is approximated as

$$g_{sploss} = \Delta g_{spideal} - \Delta g_{spactual} = \Delta g_{spactual} (1 - 1/\eta_{comb}) \quad (50)$$

Nichols¹³ has investigated the application of GSP for experimental evaluation of combustor effectiveness when both pressure drop and combustion efficiency simultaneously play a role in determining combustor performance. The results of that study indicate that GSP is a very desirable means for evaluating overall effectiveness of combustors.

If the exergy content of the fuel is approximated as its lower heating value (LHV), then exergy loss due to incomplete combustion is simply the difference between the ideal enthalpy rise across the combustor and the actual exergy rise

$$ex_{loss} \approx (f/a)(LHV)(1 - \eta_{comb}) \quad (51)$$

This can be used to create an expression for exergy transfer due to combustion efficiency by substituting Eqs. (8) and (51) into Eq. (7),

In the preceding expression, c_p is the constant pressure specific heat of the flow entering the combustor. Note that if the quantity $f/a(LHV)$ is much greater than the flow exergy entering the combustor, then Eq. (52) reduces to

$$ex_{out}/ex_{in} \approx \eta_{comb} \quad (53)$$

Note that this expression is an estimate on exergy loss due to unburned fuel only. It does not account for the destruction of exergy via nonequilibrium combustion in the combustor.

Loss Due to Shaft Power Extraction

In addition to the aerothermodynamic loss mechanisms already described, mechanical elements in a propulsion system are also sources of loss. Typical loss mechanisms are windage, bearing friction, gear train losses, and shaft power extracted to drive engine accessories. Shaft power losses are usually measured in terms of absolute horsepower required, or in terms of the ratio of loss to total shaft power input. The latter quantity will be used herein by virtue of its nondimensional nature. It is assumed that the shaft power lost due to parasitics is ultimately converted into heat, which may itself

contain some work potential, but may or may not be usable. If the heat produced is not usable, then it is simple to derive expressions for exergy, GSP, and thrust work transfer as a function of percent power loss:

$$ex_{out}/ex_{in} = g_{sp,out}/g_{sp,in} = 1 - w_{loss}/w_{in} \quad (54)$$

$$wp_{out}/wp_{in} = \sqrt{1 - w_{loss}/w_{in}} \quad (55)$$

If the heat rejected can be recovered in some useful form, then the transfer function must also include a term to account for this. The first law of thermodynamics implies that the steady-state rate of heat rejection must be equal to the parasite power required. If the waste heat is rejected at a temperature T_{reject} , then the total exergy of the waste heat stream is

$$ex_{reject} = w_{loss}(1 - T_{amb}/T_{reject}) \quad (56)$$

Appending this term to Eq. (54) yields an expression for exergy transfer as a function of percent shaft power lost:

$$ex_{out}/ex_{in} = 1 - (w_{loss}/w_{in})(T_{amb}/T_{reject}) \quad (57)$$

It is clear from Eq. (57) that the higher the rejection temperature the more exergy can be recovered. This also applies for recovery of GSP and thrust work potential, although the recovery capability will depend greatly on the specifics of any given scenario.

Comparison of Efficiency to Work Potential

Table 1 gives a summary of the various work potential transfer functions discussed in this paper. A general trend is evident in these expressions: Exergy transfer is always a function of nondimensional inlet pressure and temperature, whereas the expressions for GSP and thrust work potential are only a function of nondimensional inlet pressure. This should be no surprise given that the definition of exergy requires a reference temperature and pressure, whereas GSP requires only a reference pressure. Thrust work potential is similar to GSP in this regard, except that it additionally requires the definition of a flight velocity. Therefore, this general trend is a manifestation of the work potential definitions themselves, specifically the boundary conditions imposed by each FOM.

Each of the transfer functions given in Table 1 is essentially a form of component efficiency, each having a unique set of assumptions implicit in it. These transfer functions could be used in lieu of component efficiencies if one were so inclined. The chief advantage of the work transfer functions over the classic component efficiencies is that they are a direct measure of work transfer and are all directly comparable to one another. This does not hold for component efficiencies. The chief advantages of component efficiencies are that they are widely used and are relatively easy to measure. (They are de facto standards for measurement of component performance.) On the former point, work transfer functions are not as useful, although it is hoped that the equations in Table 1 will facilitate easy translation in the future. On the latter point, efficiencies are almost always determined via measurement of intermediate variables such as temperatures, pressures, forces, etc. Work potential transfer functions can also be calculated using this same data and so are at least on par with efficiencies in this regard.

Work transfer functions require the definition of reference conditions as a datum against which work potential is measured. This has the disadvantage of adding complications to the component performance analysis process. In particular, for flight vehicles where the ambient conditions may change considerably with flight condition, one must change the reference datum to follow the instantaneous ambient conditions if a truly accurate estimate on work potential is desired. The counterpoint is that calculations to allow for instantaneous changes in flight conditions are quite amenable to implementation in code and so do not necessarily represent an increased workload for the analyst. The advantage, of course, is that such an implementation yields a truly accurate and intuitive understanding of component performance relative to the ideal.

Table 1 Summary of expressions for loss as a function of component efficiency

Loss source	Definition	Exergy transfer function	GSP transfer function	Thrust work potential transfer function
Inlet	$\frac{P_{0,in}}{P_{0,amb}}$	$1 - \frac{\ln(\eta_R)}{[\gamma/(\gamma-1)](T_{in}/T_{amb} - 1 - \ln(T_{in}/T_{amb})) + \ln(P_{in}/P_{amb})}$	$\frac{1 - [(P_{in}/P_{amb})\eta_R]^{(1-\gamma)/\gamma}}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}}$	$\sqrt{\frac{1 - [(P_{in}/P_{amb})\eta_R]^{(1-\gamma)/\gamma}}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}}}$
Compressor	$\frac{W_{in,ideal}}{W_{in,actual}}$	$1 - \frac{\eta_c \ln\left[\frac{PR^{(1-\gamma)/\gamma}}{(T_{in}/T_{amb})} \left(\frac{PR^{(\gamma-1)/\gamma}}{(1-\eta_c)} + 1\right)\right]}{\eta_c \ln\left[\frac{PR^{(1-\gamma)/\gamma}}{(T_{in}/T_{amb})} \left(\frac{PR^{(\gamma-1)/\gamma}}{(1-\eta_c)} + 1\right)\right]}$	$\frac{\eta_c + PR^{(\gamma-1)/\gamma} - 1}{PR^{(\gamma-1)/\gamma}}$	$\sqrt{\frac{\eta_c + PR^{(\gamma-1)/\gamma} - 1}{PR^{(\gamma-1)/\gamma}}}$
Turbine	$\frac{W_{out,actual}}{W_{out,ideal}}$	$\left\{ 1 + \frac{\ln\left[\frac{PR^{(\gamma-1)/\gamma}}{(T_{in}/T_{amb})} \left(\frac{PR^{(\gamma-1)/\gamma}}{(1-\eta_T)} + \eta_T\right)\right]}{\eta_T \ln\left[\frac{PR^{(1-\gamma)/\gamma}}{(T_{in}/T_{amb})} \left(\frac{PR^{(\gamma-1)/\gamma}}{(1-\eta_T)} + \eta_T\right)\right]} \right\}^{-1}$	$\eta_T PR^{(1-\gamma)/\gamma}$	$\sqrt{\eta_T PR^{(1-\gamma)/\gamma}}$
Pressure drop	$\frac{P_{0,in} - P_{0,out}}{P_{0,in}}$	$1 - \frac{\ln(1 - \Delta P/P_{in})}{[\gamma/(\gamma-1)](T_{in}/T_{amb} - 1 - \ln(T_{in}/T_{amb})) + \ln(P_{in}/P_{amb})}$	$\frac{1 - [(P_{in}/P_{amb})^{(1-\gamma)/\gamma}]}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}}$	$\sqrt{\frac{1 - [(P_{in}/P_{amb})^{(1-\gamma)/\gamma}]}{1 - (P_{in}/P_{amb})^{(1-\gamma)/\gamma}}}$
Incomplete combustion	$\frac{Q_{out,actual}}{Q_{out,ideal}}$	$1 - \frac{[(f/a)(LHV)/c_p T_{amb}](1 - \eta_{comb})}{T_{in}/T_{amb} - 1 - \ln(T_{in}/T_{amb}) + \ln(P_{in}/P_{amb}) + (f/a)(LHV)/c_p T_{amb}}$	$\frac{(f/a)(LHV)\eta_{comb} + T_{in}c_p}{(f/a)(LHV) + T_{in}c_p}$	$\sqrt{\frac{(f/a)(LHV)\eta_{comb} + T_{in}c_p}{(f/a)(LHV) + T_{in}c_p}}$
Nozzle internal losses	$\frac{F_{G,actual}}{F_{G,ideal}}$	$1 - \frac{\ln\left[\frac{1 - C_{FG}^2(1 - T_{amb}/T_{in})}{(T_{in}/T_{amb})} - 1\right]}{\ln\left[\frac{1 - C_{FG}^2(1 - T_{amb}/T_{in})}{(T_{in}/T_{amb})} - 1\right]}$	C_{FG}^2	C_{FG}
Shaft power extraction	$\frac{H P_{loss}}{H P_{in}}$	$1 - \frac{w_{loss}}{w_{in}} \frac{T_{amb}}{T_{reject}}$	$1 - \frac{w_{loss}}{w_{in}}$	$\sqrt{1 - \frac{w_{loss}}{w_{in}}}$

Conclusions

The various comparisons of component efficiencies in terms of work potential transfer shown herein illustrate why classical models for component efficiency are not entirely satisfactory for engine analysis and design. The chief drawback of efficiency-based component performance metrics is that they are custom made using disparate definitions, are not directly comparable to one another, and give little insight as to transfer of work potential. The work potential perspective can provide a deeper understanding of transfer and loss of work potential in prime movers and so is a useful supplement to efficiency-based component performance metrics.

The component-specific analyses presented herein revealed a variety of interesting and sometimes counterintuitive results. For instance, it was shown that nozzle thrust coefficient is a work potential FOM, with GSP transfer being a limiting case for exergy loss due to nozzle internal aerodynamics. Loss due to pressure drops is a function of both inlet temperature and pressure, with the magnitude of loss decreasing precipitously as these parameters increase. Exergy transfer in a compressor actually goes to unity as compressor efficiency goes to zero, counter to what one might intuitively expect. Exergy transfer in a turbine goes to zero as adiabatic efficiency approaches zero. GSP transfer in compressors and turbines is a linear function of adiabatic efficiency. Exergy loss due to shaft parasitic losses is a function of average heat rejection temperature. All of these phenomena are not evident using only efficiency-based component performance representations. However, they are plainly obvious when viewed from a work potential perspective.

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